Incoherent accessible white-light solitons in strongly nonlocal Kerr media

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We present a theoretical investigation of incoherent accessible white-light solitons in strongly nonlocal medium with noninstantaneous Kerr nonlinearity. This soliton has elliptic Gaussian intensity profile and anisotropic spatiotemporal coherence properties. For this soliton to exist, the spatial coherence distance should be larger for lower frequencies and shorter for higher frequencies. When the incident power differs from the critical value, we demonstrate the periodic harmonic oscillations of the accessible white-light solitons.

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The observation of partially coherent solitons changed the previous concept that solitons are fully coherent entities [1]. Subsequently, self-trapping of spatiotemporally incoherent white-light solitons was observed [2]. Last year, incoherent solitons in a discrete optical lattice were also reported experimentally [3]. An experimental observation of such incoherent solitons was carried out in photorefractive crystal which has a noninstantaneous nonlinear response [1–3]. It was also shown that incoherent solitons can be obtained in nematic liquid crystals for its nonlinear response is sufficiently slow to allow for formation of incoherent solitons [4,5]. Nematic liquid crystal is also a strongly nonlocal media [6] so that experimental observation of accessible solitons [7] turns into reality [8]. Partially coherent solitons in nonlocal media have been studied extensively [9–11]. It is also shown that incoherent solitons can exist in instantaneous nonlocal nonlinear media [12]. Nonlocal white-light solitons have been discussed in the case of logarithmic nonlinearity [13], where the degree of the nonlocality is arbitrary. Yet white-light solitons have not been studied in Kerr media with extremely large nonlocality [14].

Here we present a theoretical investigation of accessible white-light solitons in strongly nonlocal media with noninstantaneous Kerr nonlinearity. These incoherent solitons have elliptic Gaussian intensity profile and anisotropic spatial correlation statistics. For this soliton to exist, the spatial coherence distance should be larger for lower frequencies and shorter for higher frequencies. The propagation of the accessible white-light solitons depends on the incident power as well as the coherence properties of the beam. When the incident power differs from the critical value, the solitons will undergo periodic harmonic oscillations. Relevant properties are discussed in detail.

Consider a spatiotemporally incoherent white-light beam propagating in a strongly nonlocal media with noninstantaneous Kerr nonlinearity. Suppose that the frequencies of the beam lie within the interval $[\omega_0(1-\Delta),\omega_0(1+\Delta)]$, where ω_0 is the central frequency and Δ denotes the width of the temporal power spectrum. The effective theory of treating the propagation of spatially and temporally incoherent light is the so-called mutual spectral density theory [15–18]. The propagation of the incoherent beam satisfies an integrodifferential equation [15]

$$\partial B_\omega/\partial z-(i/2k_\omega)[\nabla^2_{\perp 1}-\nabla^2_{\perp 2}]B_\omega = \frac{ik_\omega}{n_0}\{\delta n(I(\mathbf{r}_1,z))-\delta n(I(\mathbf{r}_2,z))\}B_\omega(\mathbf{r}_1,\mathbf{r}_2,z), \quad (1)$$

where $k_\omega=n_0\omega/c$ is the wave vector and $B_\omega(\mathbf{r}_1,\mathbf{r}_2,z)$ represents the mutual spectral density describing the correlation statistics and the intensity of the beam [15–18], $\nabla_\perp$ is a two-dimensional transverse Laplacian operator. In nonlocal Kerr media, the nonlinear part of the refractive index can be expressed as $\delta n(I)=\int R(\vec{r}-\vec{r}\,')I(\vec{r}\,',z)d\vec{r}\,'$, where $R(\vec{r}-\vec{r}\,')$ is the circularly symmetric nonlocal response function and $I(\mathbf{r},z)=1/2\pi\int_0^\infty d\omega\, B_\omega(\mathbf{r},\mathbf{r},z)$ is the time-averaged optical intensity. In the case of strong nonlocality, $\delta n(I)$ (expanded in Taylor's series to the second order) is defined as $\delta n(I)=R_0P_0-\frac{1}{2}\gamma P_0 r^2$ [11,19,21], $R_0=R(\vec{r})|_{r=0}$ is the maximum of $R(\vec{r})$, $\gamma=-R^{(2)}(\vec{r})|_{r=0}>0$, and $P_0=\int I\,dx\,dy$ is the total power. From Eq. (1), we get the propagation equation of the incoherent beam

$$i\frac{\partial B_\omega}{\partial z}+\frac{1}{2k_\omega}(\nabla^2_{\perp 1}-\nabla^2_{\perp 2})B_\omega+\frac{k_\omega}{2n_0}(r_2^2-r_1^2)\gamma P_0 B_\omega=0. \quad (2)$$

For convenience, introducing two new spatial coordinates $\vec{p}$ and $\vec{q}$

$$\vec{p}=\tfrac{1}{2}(\vec{r}_1+\vec{r}_2), \quad \vec{q}=\vec{r}_1-\vec{r}_2, \quad (3)$$

under a new spatial coordinates system, Eq. (2) turns into

$$\partial B_\omega/\partial z-(i/k_\omega)\vec{\nabla}_{\vec{p}}\cdot\vec{\nabla}_{\vec{q}}B_\omega+(ik_\omega/n_0)\gamma P_0\vec{p}\cdot\vec{q}B_\omega=0. \quad (4)$$

The solution of Eq. (4) for the mutual spectral density is in the elliptic Gaussian profile [17]

$$B_\omega(\vec{p},\vec{q})=A_\omega\exp\left[-\frac{p_x^2}{2W_{x0}^2}-\frac{q_x^2}{2Q_{x0}^2}-\frac{p_y^2}{2W_{y0}^2}-\frac{q_y^2}{2Q_{y0}^2}\right], \quad (5)$$

where A_ω denotes the spectral density, W_{x0} and W_{y0} denote the beam widths, and Q_{x0} and Q_{y0} are related to the spatial correlation distance of the beam. Equation (4) has the mathematical form that is identical to the equation for elliptic Gaussian solitons in logarithmic local nonlinear media when the incoherent optical field is propagation invariant [17,20]. Since nonlocality can prevent the catastrophic collapse of self-focused beams, allowing $(2+1)D$ solitons in Kerr-type media [21], and the two-dimensional Gaussian mutual spectral density will maintain its coherence properties during its propagation. Inserting Eq. (5) into Eq. (4), we obtain the beam parameters that should obey

$$Q_{j0} = c/\omega W_{j0}\sqrt{n_0 \gamma P_0} = (\omega_0/\omega)Q_0, \quad j = x,y, \tag{6}$$

where $Q_0 = c/(\omega_0 W_{j0}\sqrt{n_0\gamma P_0})$ is associated with the spatial correlation distance at frequency ω_0 and c is the speed of the light. From Eq. (6), we find that for this elliptic incoherent soliton to exist, the spatiotemporal coherence properties should be anisotropic such as the incoherent solitons in local media [22,23]. The quantities Q_{j0} are related to the total power so that the nonlinearity in strongly nonlocal Kerr media depends on the incident power, whereas in other media the coherence properties of incoherent solitons are only related to the spatial distribution of the intensity [9,17,13]. The spatial coherence properties for each frequency constituent can be found from the complex coherence factor [24], combining Eqs. (5) and (6) we obtain

$$\mu_\omega(r_1,r_2,z) = B_\omega(r_1,r_2,z)/\sqrt{B_\omega(r_1,r_1,z)B_\omega(r_2,r_2,z)} = \prod_{j=x,y} \exp\left\{\left[\frac{1}{4W_{j0}^2} - \frac{\omega^2 W_{j0}^2 n_0 \gamma P_0}{c^2}\right]\frac{q_j^2}{2}\right\}. \tag{7}$$

The spatial coherence distance can be obtained from the following expression [24]:

$$l_j(\omega) = \int_{-\infty}^{+\infty} |\mu_\omega(r_{1j},r_{2j},z)|^2 dr_{2j} = (2\sqrt{\pi}cW_{j0}/\sqrt{4\omega^2 W_{j0}^4 n_0\gamma P_0 - c^2}), \quad j = x,y, \tag{8}$$

where $l_x(\omega)$ and $l_y(\omega)$ are the spatial coherence distance at frequency ω in the x and y directions. Equation (8) is the existence curve for the white-light solitons in strongly nonlocal media with a Kerr nonlinearity. In Fig. 1 we plot the existence curve and the complex coherence factors at three frequencies of such accessible white-light solitons. We assume the nonlocal response function, of the media is in the Gaussian form [21]

$$R(\vec{r}) = 1/2\pi\sigma^2 \exp[-(x^2+y^2)/2\sigma^2] \tag{9}$$

which implies that $\gamma = 1/2\pi\sigma^4$; here σ is the width of the response function which is much larger than the beam width in strongly nonlocal media.

From the existence curve we find that for white-light solitons to exist in strongly nonlocal media, the spatial correlation distance should be larger for lower frequencies and smaller for higher frequencies at a given incident power; this result has been studied in other papers [13,16–18]. It is also shown that the spatial coherence distance will decrease when the total power increases. In strongly nonlocal media, the nonlinear part of the refractive index depends on the incident power [19,21]. When the total power increases, the tendency of self-trapping will strengthen. To balance this, it requires that the beam should be more incoherent, i.e., the coherence distance decreases. From Fig. 1(b) we find that the width of the complex coherence factor is larger for lower frequencies and smaller for higher frequencies, which implies that the spatial correlation distance is larger for lower frequencies and smaller for higher frequencies. The width of the complex coherence factors will also decrease when the incident power increases.

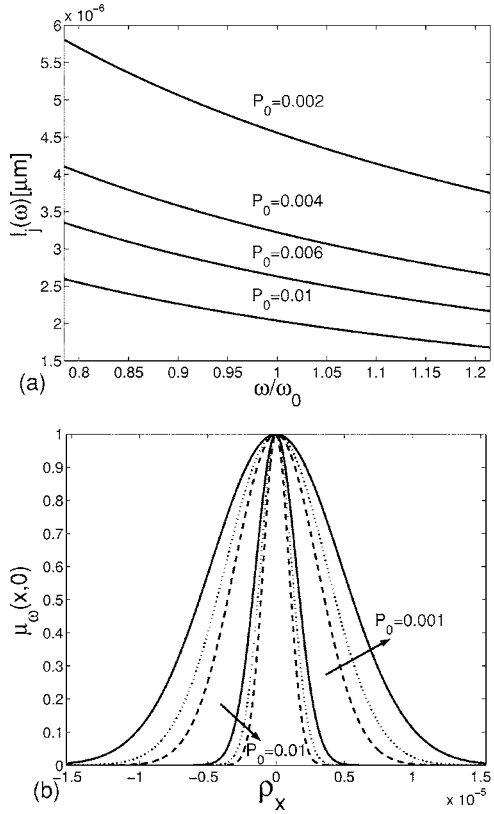

FIG. 1. The existence curve and the complex coherence factors of accessible white-light solitons. The initial parameters are $n_0 = 2.3$, $W_{j0} = 10\ \mu$m, $\gamma = 2.5\times 10^9$ $(\sigma = 283 W_{j0})$, and $\omega_{min} = 2.69 \times 10^{15}$ Hz (solid line), $\omega_0 = 3.44\times 10^{15}$ Hz (dotted line), and $\omega_{max} = 4.19\times 10^{15}$ Hz (dashed line) for (b).

From Eq. (8) it follows that the beam widths should be larger than a threshold value, i.e., $W_{j0} > \sqrt{c/(2\omega\sqrt{n_0\gamma P_0})}$. This threshold must be satisfied for every frequency within the spectrum. Suppose the temporal power spectrum of the beam is $[\omega_{min},\omega_{max}]$; then the beam widths should obey the relation $W_{j0} > \sqrt{c/[2\omega_0(1-\Delta)\sqrt{n_0\gamma P_0}]}$.

In the above section, we have obtained the existence curve for stationary accessible white-light solitons [Eqs. (5) and (6)]. Next we will analyze the periodic harmonic oscillations of such incoherent solitons. According to Eq. (6), assume that the mutual spectral density is in the following form during its propagation [13,17]

$$B_\omega(p_x,q_x,p_y,q_y,z) = A_\omega(z)\prod_{j=x,y}\exp\left[-\frac{p_j^2}{2W_j^2(z)} - \frac{q_j^2}{2S_j^2(z)}\frac{\omega^2}{\omega_0^2} + ip_j q_j \phi_\omega(z)\right], \tag{10}$$

where $A_\omega(z)$ and $\phi_\omega(z)$ denote the amplitude and the phase of the mutual spectral density, $W_j(z)$ is the beam width, and

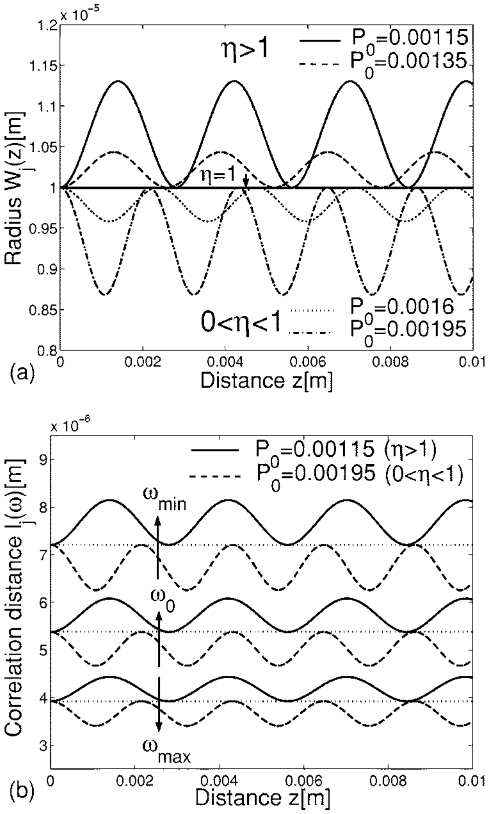

FIG. 2. Harmonic oscillations of the beam width and the coherence radius when the incident power differs from the critical value ($\eta \neq 1$). The initial parameters are $n_0=2.3$, $W_j(0)=10\ \mu$m, $\gamma=2.5 \times 10^9$ [$\sigma=283W_j(0)$], $S_j(0)=3\ \mu$m, the critical value of the power is $P_c=0.001\,46$ (W) and $\omega_{min}=2.19\times10^{15}$ Hz, $\omega_0=3.44\times10^{15}$ Hz, and $\omega_{max}=4.69\times10^{15}$ Hz.

$S_j(z)$ is associated with the spatial correlation distance at frequency ω_0. Inserting expression (10) into Eq. (4), we obtain a set of ordinary differential equations for the parameters of the mutual spectral density

$$\frac{dA_\omega(z)}{dz} = -\frac{2}{k_\omega}\phi_\omega(z)A_\omega(z), \tag{11}$$

$$\frac{dW_j(z)}{dz} = \frac{1}{k_\omega}\phi_\omega(z)W_j(z), \tag{12}$$

$$\frac{dS_j(z)}{dz} = \frac{1}{k_\omega}\phi_\omega(z)S_j(z), \tag{13}$$

$$\frac{1}{k_\omega}\frac{d\phi_\omega(z)}{dz} = \frac{1}{k_{\omega_0}^2}\frac{1}{S_j^2(z)W_j^2(z)} - \frac{\phi_\omega^2(z)}{k_\omega^2} - \frac{\gamma P_0}{n_0}. \tag{14}$$

From Eqs. (12) and (13) we obtain the relation $S_j(z)/W_j(z) = S_j(0)/W_j(0)$, which shows the coherence properties of the beam during the evolution. Combining Eqs. (11) and (12) gives the amplitude $A_\omega(z)=W_j^2(0)/W_j^2(z)A_\omega(0)$. Finally, inserting Eq. (14) into Eq. (12), we obtain the evolution equation of the incoherent beam,

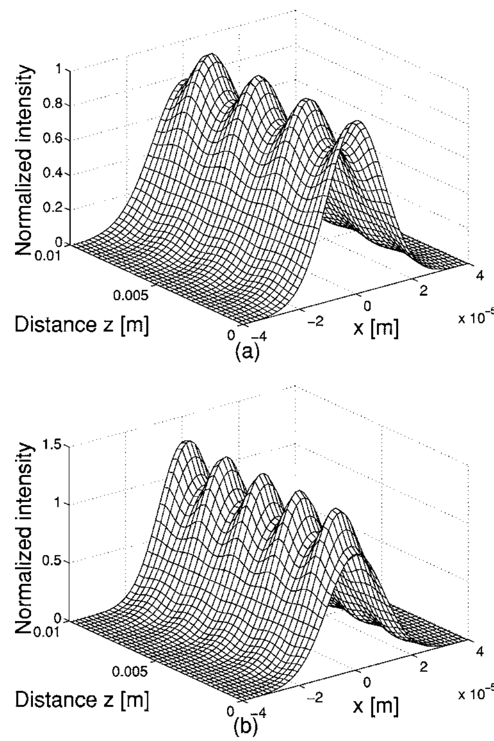

FIG. 3. Evolution properties of the normalized intensity when the solitons undergo periodic harmonic oscillation, $W_x(0)=10\ \mu$m, other parameters are as same as in Fig. 2. (a) $P_0=1.15$ mW ($\eta > 1$), (b) $P_0=1.95$ mW ($0<\eta<1$).

$$\frac{d^2W_j(z)}{dz^2} - \frac{1}{k_{\omega_0}^2}\frac{W_j^2(0)}{W_j^3(z)S_j^2(0)} + \frac{\gamma P_0}{n_0}W_j(z) = 0. \tag{15}$$

In Eq. (15) by setting $W_j(z)=W_j(0)$, we obtain the critical power $P_c=c^2/[n_0\gamma\omega_0^2W_j^2(0)S_j^2(0)]$ of the accessible white-light solitons. When the total power satisfies the critical value, the soliton will maintain its width. Otherwise, solitons will undergo periodic oscillation. Assume that the beam at $z=0$ has $dW_j(z)/dz|_{z=0}=0$; integrating Eq. (15) once, we can obtain

$$[dW_j(z)/dz]^2 + [1/k_{\omega_0}^2 S_j^2(0)][W_j^2(0)/W_j^2(z) - 1] + (\gamma P_0/n_0)[W_j^2(z) - W_j^2(0)] = 0. \tag{16}$$

Let $W_j(z)/W_j(0)=Y(z)$ and rewrite Eq. (16) as

$$(dY/dz)^2 = m(Y^2-1)(\eta - Y^2)/Y^2W_j^2(0), \tag{17}$$

where $\eta=P_c/P_0=c^2/[n_0\gamma P_0\omega_0^2W_j^2(0)S_j^2(0)]$ and $m=\gamma P_0 W_j^2(0)/n_0$. From Eq. (17) we get the simple form of accessible white-light solitons in strongly nonlocal media,

$$W_j^2(z) = W_j^2(0)[\cos^2(\beta z) + \eta\sin^2(\beta z)], \tag{18}$$

where $\beta=\sqrt{m}/W_j(0)=\sqrt{\gamma P_0/n_0}$. When $\eta=1$, $W(z)=W_0$, the accessible white-light solitons maintain their width during the propagation. When $\eta\neq 1$, the solitons will undergo periodic harmonic oscillation [7]. Combining Eqs. (8) and (10), we can obtain the coherence distance of the incoherent light beam during propagation,

$$l_j(\omega) = 2\sqrt{\pi}S_j(0)W_j(z)/\sqrt{4W_j^2(0)\omega^2/\omega_0^2 - S_j^2(0)}. \quad (19)$$

In Fig. 2 we show the linear harmonic oscillations of the beam width and the coherence radius when the accessible white-light solitons undergo periodic oscillations. Figure 2(a) shows that the beam will oscillate periodically between $W_j(0)$ and somewhat larger values when the power is smaller than the critical value ($\eta > 1$). The beam width will increase at first and decrease when it reaches the maximum. The amplitudes and the periods of the oscillation are larger for lower incident power and smaller for higher incident power. When the power is larger than the critical value ($0 < \eta < 1$), the beam width will oscillate between $W_j(0)$ and somewhat smaller values. The amplitudes of the oscillation will increase while the periods of the oscillation will decrease when the incident power increases. Figure 2(b) shows the oscillation behaviors of the coherence distance at three frequencies. We can see that the periodic oscillations of all frequencies is the same at a given power, which is the same as the periodic oscillations of the beam width. The coherence distance is larger for lower frequencies and smaller for higher frequencies at an incident power. When the incident power is larger than the critical value, the correlation distance of all frequency constituents will oscillate between their initial values and somewhat smaller values. When the incident power is smaller than the critical value, the correlation distance of all frequency constituents will oscillate between their initial values and somewhat larger values. The oscillation periodic will decrease when the power increases for any conditions.

The intensity of the white-light beam (consider only x direction, i.e., at the cross section $y=0$) is obtained from Eq. (10) during the propagation process through

$$I(x,z) = [I_0 W_x^2(0)/W_x^2(z)] \exp\left[-x^2/2W_x^2(z)\right], \quad (20)$$

where I_0 is the time-averaged peak intensity of the incident beam. In Fig. 3 we plot the evolution properties of the normalized intensity when the solitons undergo a periodic harmonic oscillation. When the incident power is smaller than the critical power ($\eta > 1$), the normalized intensity peak of the incoherent beam decreases first and oscillates between 1 and a somewhat smaller value, as shown in Fig. 3(a). When the incident power is smaller than the critical power ($0 < \eta < 1$), the normalized intensity peak of the incoherent beam increases first and oscillates between 1 and a somewhat larger value, as shown in Fig. 3(b). The intensity decreases when the beam expands and increases when the beam contracts.

In summary, incoherent white-light solitons in strongly nonlocal media with a Kerr nonlinearity are studied. For these solitons to exist, the coherence distance should be larger for lower frequencies and shorter for higher frequencies. The propagation of the incoherent accessible white-light solitons depends on the incident power as well as the coherence properties of the beam. When the incident power differs from the critical value, the solitons will undergo periodic harmonic oscillations.

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[1] M. Mitchell, Z. Chen, M. Shih, and M. Segev, Phys. Rev. Lett. **77**, 490 (1996).
[2] M. Mitchell and M. Segev, Nature (London) **387**, 880 (1997).
[3] O. Cohen, G. Bartal, H. Buljan, T. Carmon, J. W. Fleischer, M. Segev, and D. N. Christodoulides, Nature (London) **433**, 500 (2005).
[4] M. Peccianti and G. Assanto, Opt. Lett. **26**, 1791 (2001).
[5] M. Peccianti and G. Assanto, Phys. Rev. E **65**, 035603(R) (2002).
[6] C. Conti, M. Peccianti, and G. Assanto, Phys. Rev. Lett. **91**, 073901 (2003).
[7] A. Snyder and J. Mitchell, Science **276**, 1538 (1997).
[8] C. Conti, M. Peccianti, and G. Assanto, Phys. Rev. Lett. **92**, 113902 (2004).
[9] W. Krolikowski, O. Bang, and J. Wyller, Phys. Rev. E **70**, 036617 (2004).
[10] K. G. Makris, H. Sarkissian, D. N. Christodoulides, and G. Assanto, J. Opt. Soc. Am. B **22**, 1371 (2005).
[11] M. Shen, Q. Wang, J. Shi, P. Hou, and Q. Kong, Phys. Rev. E **73**, 056602 (2006).
[12] O. Cohen, H. Buljan, T. Schwartz, J. W. Fleischer, and M. Segev, Phys. Rev. E **73**, 015601(R) (2006).
[13] M. Shen, Q. Wang, J. Shi, Y. Chen, and X. L. Wang, Phys. Rev. E **72**, 026604 (2005).
[14] C. Rotschild, O. Cohen, O. Manela, M. Segev, and T. Carmon, Phys. Rev. Lett. **95**, 213904 (2005).
[15] H. Buljan, A. Siber, M. Soljacic, and M. Segev, Phys. Rev. E **66**, 035601(R) (2002).
[16] H. Buljan, M. Segev, M. Soljacic, N. K. Efremidis, and D. N. Christodoulides, Opt. Lett. **28**, 1239 (2003).
[17] H. Buljan, A. Siber, M. Soljacic, T. Schwartz, M. Segev, and D. N. Christodoulides, Phys. Rev. E **68**, 036607 (2003).
[18] H. Buljan, T. Schwartz, M. Segev, M. Soljacic, and D. N. Christodoulides, J. Opt. Soc. Am. B **21**, 397 (2004).
[19] Q. Guo, B. Luo, F. Yi, S. Chi, and Y. Xie, Phys. Rev. E **69**, 016602 (2004).
[20] W. Krolikowski, D. Edmundson, and O. Bang, Phys. Rev. E **61**, 3122 (2000).
[21] O. Bang, W. Krolikowski, J. Wyller, and J. J. Rasmussen, Phys. Rev. E **66**, 046619 (2002).
[22] E. D. Eugenieva, D. N. Christodoulides, and M. Segev, Opt. Lett. **25**, 972 (2000).
[23] O. Katz, T. Carmon, T. Schwartz, M. Segev, and D. N. Christodoulides, Opt. Lett. **29**, 1248 (2004).
[24] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, New York, 1995).